Randomized Routing as a Regularized Solution to the Route Cost Minimization Problem

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ABSTRACT

Often a link-state routing takes a form of the cost based scheme which admits an arriving request on the minimum cost route if this cost does not exceed the cost of the request, and rejects the request otherwise. Cost based strategies naturally arise as a result of optimization of the network performance or incorporating Quality of Service (QoS) requirements into the admission and routing processes. In the former case the implied cost of the resources represents expected future revenue losses due to insufficient resources for servicing future requests. In the latter case the cost of a route represents the expected level of QoS , e.g., bandwidth, delay, packet loss, etc., provided to the request carried on this route. In both cases due to the aggregation, statistical nature of the resource costs, delays in disseminating signaling information, non-steady or adversarial operational environment the cost of the resources may not be known exactly. Usually this uncertainty is modeled by assuming that resource costs are random variables with fixed probability distributions. We propose to explicitly recognize that the minimum cost route selection as an ill-posed problem and to view randomized routing as a regularized solution to this problem. We consider a specific case of regularization intended to guard against adversarial uncertainty, i.e., worst case scenario, with respect to the resource costs lying within known "confidence" intervals. Assuming that the network minimizes and the adversarial environment maximizes the losses resulted from non-optimal admission and routing decisions due to the uncertainty, we identify the optimal admission and routing decisions with the Nash equilibrium strategy in the corresponding game. We explicitly identify this strategy in a case of parallel, homogeneous structure.

Keywords: randomized routing, minimum cost routing, uncertainty, regularization, games, Nash equilibrium.

1. INTRODUCTION

Often a link-state routing takes a form of the cost based scheme which admits an arriving request on the minimum cost route if this cost does not exceed the cost of the request, and rejects the request otherwise. Cost based strategies naturally arise as a result of optimization of the network performance [1] or incorporating Quality of Service (QoS) requirements into the admission and routing processes [2]. In the former case the implied cost of the resources represents expected future revenue losses due to insufficient resources for servicing future requests. In the latter case the cost of a route represents the expected level of QoS , e.g., bandwidth, delay, packet loss, etc., provided to the request carried on this route. In both cases due to the aggregation, statistical nature of the resource costs, delays in disseminating signaling information, non-steady or adversarial operational environment the cost of the resources may not be known exactly.

This uncertainty does not affect the routing decisions if the minimum cost route can be identified with required degree of confidence. The problem, however, is that due to the very nature of the minimum cost routing, the minimum cost route typically *cannot* be identified with a reasonable degree of confidence. Indeed, except for some anomalies, the cost of a link is an increasing function of the link load. Minimum cost routing increases the load carried on a minimum cost route until admission control takes over, or at least two routes have the same cost. This positive feedback attempts to equalize the costs of several routes with each other and with the cost of a request, and may cause oscillations in the optimal route selection, which are perceived as route flapping phenomenon when the frequency of optimal route updates significantly increases.

Usually, uncertainty in the resource costs is modeled by assuming that the resource costs are random variables with fixed probability distributions, which may or may not be known to the network [2]. From decision theoretic perspective this approach lies within Bayesian framework [3]. However, even when the forms of the probability distributions can be reliably identified, e.g., exponential distributions for the delays, the parameters of these distributions, e.g., average delays, remain to be subject to uncertainty within the corresponding confidence regions, leaving the problems of sensitivity and instability unresolved. Empirical results suggest that *performance/robustness* curve of the routing algorithm can be often improved by allowing randomization of the routing decisions [4]. However, Bayesian framework resulted in deterministic optimal strategies does not allow one to support and quantify this claim.

This paper proposes to tackle the problem of uncertainty in the resource costs by viewing route cost minimization as an ill-posed problem [5]. The standard technique for solving ill-posed problem is regularization, i.e., penalizing solution for sensitivity to the variable contaminated by noise. We propose to view randomized routing as a regularized solution to the route cost minimizatio problem, where regularization is based on the "confidence region" $c \in C$ rather than point estimates $c \approx \tilde{c}$ for the vector $c = (c_l)$ of the link costs c_l. In a situation when link costs lie within certain confidence region, the set of "acceptable" routes includes all feasible routes, which may be optimal, given the constraints imposed by the confidence interval. We propose to regularize solution to the routing optimization problem by randomizing routing selection within the optimal set of routes. Different regularization techniques lead to different randomization among acceptable routes. This framework can be extended to a situation of random link costs with probability distributions containing uncertain parameters.

We consider a specific case of regularization intended to guard against adversarial uncertainty, i.e., the worst case scenario, with respect to the resource costs lying within known confidence regions. Assuming that the network minimizes and the adversarial environment maximizes the losses resulted from non-optimal admission and routing decisions due to the uncertainty, we identify the optimal routing with Nash equilibrium strategy in the corresponding game. We explicitly identify the corresponding Nash equilibrium routing strategy in a case of parallel, homogeneous structure under various scenarios for the adversarial environment. The game theoretic framework approach to network management under uncertainty has been proposed in [6] and then applied to cost based admission control for a case of a single feasible route in [7].

Various assumptions on the capabilities of the adversarial environment to manipulate the link costs produce different game models. For example, in a case of extremely omnipotent environment, capable of synchronized selection of the link cost for all links, the adequate game model includes two players: the network and the environment. In a case when link costs are selected independently for different links, the adequate game model is a non-cooperative game of $1+L$ players where the number of links in the network is L. In this game model one player representing the network is trying to minimize losses and other L players representing network links are trying to maximize losses. Various intermediate scenarios with respect to synchronization of link costs are possible.

Optimization algorithms, assuming exact knowledge of the link states, solve the problem of routing instability by splitting traffic with the same origin-destination among routes of the same cost. These algorithms are based on rerouting of the infinitesimal portions of the load, and take form of iterative process [8] or discontinuous differential equations describing sliding modes [9]. The obvious difficulty of applying these algorithms is that their stability is guaranteed only asymptotically as load granularity decreases. Even more important is that the optimal traffic split seems to be very sensitive to the unavoidable uncertainties in the fluctuations in the rates of incoming traffic. Randomization of the routing decisions can be viewed as an attempt to obtain stable traffic split.

The paper is organized as follows. Section 2 describes a minimum cost routing and its randomized version. Section 3 characterizes losses and risks associated with the admission and routing decisions under uncertainty. Section 4 describes game theoretic framework for risk management. Section 5 explicitly identifies the optimal admission strategy by solving the corresponding game in a case of parallel homogeneous structure. Finally, conclusion briefly summarizes results.

2. ROUTING

Minimum Cost Routing: Deterministic Link Costs

In a link-state model a network topology database keeps state information about nodes and links in the network $x=(x_l)\in X$ where x_l is the vector characterizing the state of a link l and adjacent to this link nodes. Often the result of selecting route r from the set of feasible routes $F=\{r_1,...,r_K\}$ can be characterized by some utility function $u_r=u(x_r)$ where vector $x_r=(x_l : l\in r)$ characterizes the state of links $l\in r$. Assuming that rejection of a request has utility zero, the definition of the utility function can be extended as follows: $u_\varnothing=0$, where $r=\varnothing$ means that the request is rejected. Given the network state x, the optimal routing strategy chooses control action that yields the maximum utility:

$$r_{opt}=\arg\max_{r\in\{\varnothing,F\}} u_r \tag{1}$$

Often link state routing takes form of the cost based scheme with utility function

$$u_r=\begin{cases} w-c_r & if \quad r\neq\varnothing \\ 0 & if \quad r=\varnothing \end{cases} \tag{2}$$

where the cost of an arriving request is w, and the cost of a route is c_r. Usually the route cost is assumed to be additive:

$$c_r=\textstyle\sum_{l\in r} c_l \tag{3}$$

where the cost of a link l is c_l. For example, network performance oriented minimum cost schemes weight the revenue brought by the arriving request w against the expected future revenue losses c_r due to insufficient resources for servicing future requests resulted from tying up certain bandwidth on links $l\in r$ [1]. In this case c_l is the implied cost of a link l. The utility (2) is the surplus value i.e., the difference between the revenue brought by the admitted request w and the implied costs of the occupied resources c_r. Maximization (1) of the utility function (2) results in the following admission and routing strategy

$$r_{opt}=\begin{cases} r_* & if \quad c_*\leq w \\ \varnothing & if \quad c_*>w \end{cases} \tag{4}$$

where the route r_* of minimum cost c_* is determined by

$$c_*=c_{r_*}=\min_{r\in F} c_r \tag{5}$$

The joint problem of rate control and routing based on resource pricing [10] can be also presented in form (1)-(2). Assuming that the user s utility of transmission at rate y is $U_s(y)$, and the incremental price of occupying resources on route r is c_r, a reasonable user s is expected to choose his transmission rate y and route $r\in F$ by solving the following individual optimization problem [10]:

$$\max_{r\in F,\, y\geq 0}\{U_s(y)-c_r y\} \tag{7}$$

Optimization problem (7) can be solved as a two step procedure with first, finding minimum cost route (1)-(2), and, then, determining the optimal transmission rate $y=y_s^{opt}$ by solving equation $dU_s/dy=c_*$.

Minimum Cost Routing: Random Link Costs

Assuming that the state of the network $x=(x_l)\in X$ is a random variable with given probability distribution $P(x)$, the optimal routing decision yields the maximum average utility [2]:

$$r_{opt}=\arg\max_{r\in\{\varnothing,F\}} E[u_r] \tag{8}$$

For the cost based scheme (2), the average utility is

$$E[u_r]=\begin{cases} w-E[c_r] & if \quad r\neq\varnothing \\ 0 & if \quad r=\varnothing \end{cases} \tag{9}$$

In a case of additive cost function (5), the optimal route selection (8) is based on the average link costs. In a case of QoS routing [2] the utility function is

$$u_r = \begin{cases} j\,(w - c_r) & if \quad r \neq \varnothing \\ 0 & if \quad r = \varnothing \end{cases} \tag{10}$$

where w characterizes the minimum level of the QoS acceptable for the arriving request, and c_r represents the level of QoS, e.g., bandwidth, delay, packet loss, etc., provided to the request carried on this route. Typically nonlinear, increasing and concave function $j\,(x)$ represents the *user* utility of receiving QoS $w + x$, while the required level of QoS is w [2]. Function $j\,(x)$ allows one to describe the user "soft" QoS requirements. A particular case of user "hard" QoS requirements corresponds to the following specific selection of the function $j\,(x)$:

$$j\,(x|w) = \begin{cases} w & if\, x > 0 \\ -\infty & if\, x < 0 \end{cases} \tag{11}$$

with some positive constant $w > 0$. Note computational difficulties associated with solving optimization problem (8) in a case of non-linear function $j\,(x)$

The following parameterized family of functions provides convenient approximation for the utility function $j\,(x)$:

$$j\,(x|w,g) = w\left(1 - e^{-gx}\right) \tag{12}$$

where $w > 0$ and $g > 0$ are some parameters. Function (12) is monotonously increasing, concave in x for any $(w,g) \in (0,\infty)^2$. When $w \to \infty$, $g \to 0$, $wg = b = const$, family (12) yields a linear utility function $j\,(x|w,g) = bx$. When $w = const$, $g \to \infty$, family (12) yields utility function (11).

Randomized Routing

Consider a situation when route $r \in \{\varnothing, F\}$ is selected with probability a_r, where the admission probability is $a = 1 - a_\varnothing$, and

$$\sum a_r = 1,\; a_r \geq 0,\; \forall r \in \{\varnothing, F\} \tag{13}$$

The expected utility of this randomized routing decision is

$$\bar{u}(a) = \sum\nolimits_{r \in F} a_r E[u_r] \tag{14}$$

where the average utility of selecting route $r \in F$ is $E[u_r]$. Maximization of the expected utility (14) with respect to the distribution $a = (a_r)$

$$\max_a \bar{u}(a, c) \tag{15}$$

subject to constraints (13) yields degenerative solution $a_r \in \{0,1\}$ which describes the optimal deterministic routing decision based on maximization of the average utility (8). In a case of deterministic route costs $c = (c_r)$, this procedure results in routing (1). Optimal solution to (15) is a discontinuous function of vector c on the hyper planes where two or more routing decisions are optimal. Since in practical applications vector c is contaminated by some noise, optimization problems (8) as well as (15) are ill-posed.

The standard technique for solving an ill-posed problem is regularization, i.e., penalizing solution a for sensitivity to contaminated variable c [5]. In a case of a minimum cost routing (10) it is natural to assume that the vector of link costs $c = (c_l)$ can be more reliably represented by the "confidence region" $c \in C$ than the point estimate $c \approx \tilde{c}$. Define the set of "acceptable" routing decisions $r \in F_* \subseteq \{\varnothing, F\}$ to be the set of all feasible routing decisions that may be optimal, given the confidence regions for the link costs. Reasonable randomized routing strategies assign positive probabilities to an acceptable routing decision, and zero probability to an unacceptable routing decision:

$$a_r > 0 \;\text{ if }\; r \in F_* \tag{16}$$

$$a_r = 0 \;\text{ if }\; r \notin F_* \tag{17}$$

This regularization framework can be easily extended to a case of random link costs by assuming that the form of the distribution for the vector c is known, but the parameters, e.g., moments, are subject to uncertainty within known "confidence regions". In the rest of the paper we consider a game theoretic framework which leads to a specific selection of probabilities a_r satisfying conditions (16)-(17).

3. LOSSES AND RISKS

Losses

The losses resulted from non-optimal network admission and routing decisions due to the uncertain link costs $c = (c_l)$ can be quantified by the following loss function [6]-[7]:

$$L(c,r) = \max_{r' \in \{\varnothing, F\}} u_{r'}(c) - u_r(c) \tag{18}$$

Combining (18) with (10) we obtain the following expression for the loss function:

$$L(c,r) = \begin{cases} \max\{0, j\,(w - c_*)\} - j\,(w - c_r) & if\; r \neq \varnothing \\ \max\{0, j\,(w - c_*)\} & if\; r = \varnothing \end{cases} \tag{19}$$

where c_* is given by (5). Function $L(x,r)$ possesses the following properties:

$$L(c,r) = 0 \;\text{ for }\; r = r_{opt} \;\text{ and }\; \forall c \in C$$

$$L(c,r) \geq 0 \;\text{ for }\; \forall (c,r) \in C \otimes \{\varnothing, r_k\}$$

for any set of the vectors of link costs $\forall c \in C$. Thus,

$$\breve{L}_{\max} = \max_{c \in C} \min_{r \in (\varnothing, F)} L(c,r) \equiv 0$$

$$\hat{L}_{\min} = \min_{r \in (\varnothing, F)} \max_{c \in C} L(c,r) \geq 0$$

Network decision to admit a request on some feasible route $r \in F$ exposes the network to potential losses $L^{adm}(c) + L^{rtn}(c,r)$ due to non-optimality of these decisions. Network decision to reject a request exposes the network to potential losses $L^{rej}(c)$ due to non-optimality of the rejection decision. Loss (19) can be expressed as follows:

$$L(c,r) = \begin{cases} L^{adm}(c) + L^{rtn}(c,r) & if\; r \neq \varnothing \\ L^{rej}(c) & if\; r = \varnothing \end{cases} \tag{20}$$

Rejection, admission and routing losses in (20) are uniquely identified as follows:

$$L^{rej}(c|w) = \max\{0, j(w - c_*)\} \quad (21)$$

$$L^{adm}(c|w) = -\min\{0, j(w - c_*)\} \quad (22)$$

$$L^{rtn}(c, r|w) = \begin{cases} j(w - c_*) - j(w - c_r) & if\ r \neq \varnothing \\ 0 & if\ r = \varnothing \end{cases} \quad (23)$$

Expressions (21)-(23) follow from (19)-(20) and the fact that $L^{rtn}(c, r_*) \equiv 0$, i.e., route selection (5) does not cause any loss. Since

$$L^{rtn}(c, r) = L^{rej}(c) - L^{adm}(c) - j(w - c_r),$$

the total loss (19) can be also rewritten as follows:

$$L(c, r) = \begin{cases} L^{rej}(c) - j(w - c_r) & if\ r \neq \varnothing \\ L^{rej}(c) & if\ r = \varnothing \end{cases}$$

It is easy to see that the rejection loss (21) is monotonously decreasing function and admission risk (22) is monotonously increasing function with respect to the partial ordering of the vector of route costs $c = (c_r : r \in F)$. Also note that

$L^{adm}(c|w) = 0$ and $L^{rej}(c|w) > 0$ if $c_* < w$

$L^{adm}(c|w) > 0$ and $L^{rej}(c|w) = 0$ if $c_* > w$

Routing risk is invariant to transformation $c_r \to c_r + a$: $L^{rtn}(c, r|w) = L^{rtn}(c + a, r|w)$. In a case of separable uncertainty in the link costs

$$c_l \in [\breve{c}_l, \hat{c}_l] \quad (24)$$

it is natural to introduce binary variables $d_l \in \{0,1\}$ such that

$$c_l = (1 - d_l)\breve{c}_l + d_l \hat{c}_l \quad (25)$$

and consider the following binary loss function of the binary vector $d = (d_l)$:

$$S(d, r) = L(c, r)\Big|_{c_l = (1-d_l)\breve{c}_l + d_l \hat{c}_l} \quad (26)$$

Substituting (25) into the right-hand side of (21)-(23) one easily obtains the binary rejection, admission, and routing loss functions.

Risks

Consider a case when the costs of links c_l are mutually jointly independent random variables with probability distributions $p_l(c|q_l)$, which depend on some parameters $q_l \in \Theta_l$:

$$P(c|q) = \prod_{l \in L} p_l(c_l|q_l) \quad (27)$$

where $c = (c_l)$, $q = (q_l)$. Averaging loss function (19) over probability distribution (27), we obtain the following risk function, which characterizes the risks resulted from non-optimal network admission and routing decisions due to the uncertain parameters $q_l \in \Theta_l$:

$$R(q, r) = E_P[L(c, r)] = \int L(c, r) P(dc|q) \quad (28)$$

Using (20)-(23) it is possible to separate total risk function (28) into rejection, admission, and routing risk functions. Note that for general topology network and arbitrary distribution (27) evaluation of the risk functions is a difficult task. Consider a particular case of parallel structure when different routes do not overlap, and link costs distributed exponentially. In this case the cost of a route and the minimum cost of a feasible route are also distributed exponentially:

$$p_r(c|\bar{c}_r) = 1 - \exp(-c/\bar{c}_r)$$

where $\bar{c}_r = \bar{c}_r(q)$ is the average cost of a route r, and

$$p_*(c|\bar{c}) = 1 - \exp(-c/\bar{c}_*)$$

where the average cost of the minimum cost route $\bar{c}_*$ is determined by the following equation: $1/\bar{c}_* = \sum(1/\bar{c}_r)$, $r \in F$. It is easy to verify that in this case the rejection, admission and routing risk functions take the following forms:

$$R^{rej} = \frac{e^{-w/\bar{c}_*}}{\bar{c}_*} \int_0^w j(x) e^{x/\bar{c}_*} dx$$

$$R^{adm} = -\frac{e^{-w/\bar{c}_*}}{\bar{c}_*} \int_{-\infty}^0 j(x) e^{x/\bar{c}_*} dx$$

$$R^{rtn} = \frac{e^{-w/\bar{c}_*}}{\bar{c}_*} \int_{-\infty}^w j(x) e^{x/\bar{c}_*} dx - \frac{e^{-w/\bar{c}_r}}{\bar{c}_r} \int_{-\infty}^w j(x) e^{x/\bar{c}_r} dx$$

In a case of utility function (12), these risk functions can be calculated explicitly:

$$R^{rej} = w\left[1 + \frac{1}{1 - g\bar{c}_*}\left(g\bar{c}_* e^{-w/\bar{c}_*} - e^{-wg}\right)\right],\ g\bar{c}_* < 1$$

$$R^{adm} = w\frac{g\bar{c}_*}{1 - g\bar{c}_*} e^{-w/\bar{c}_*},\ g\bar{c}_* < 1$$

$$R^{rtn} = wg\frac{\bar{c}_r - \bar{c}_*}{(1 - g\bar{c}_r)(1 - g\bar{c}_*)} e^{-wg},\ g\bar{c}_r < 1$$

and in a case of linear utility function $j(x) = x$ these risk functions take the following simple forms:

$$R^{rej} = w - \bar{c}_*\left(1 - e^{-w/\bar{c}_*}\right)$$

$$R^{adm} = \bar{c}_* e^{-w/\bar{c}_*}$$

$$R^{rtn} = \bar{c}_r - \bar{c}_*$$

Note that if uncertainty is separable with respect to parameters $q_l \in [\breve{q}_l, \hat{q}_l]$, it is useful to consider the binary risk function analogous to binary loss function (26).

4. GAME THEORETIC FRAMEWORK

Depending on the ability of the adversarial environment to coordinate selections of the link costs for different links, various scenarios for uncertainty are possible. We consider two extreme cases of completely centralized and completely decentralized adversarial environment. Due to space constraints we only consider a case of deterministic, but uncertain link costs (24). It can be shown that due to concavity of function j the optimal strategy for the adversarial environment assigns non-zero probabilities only to low and upper boundaries $c_l = \breve{c}_l$ and $c_l = \hat{c}_l$ of the feasible intervals $c_l \in [\breve{c}_l, \hat{c}_l]$. This allows us to deal only with binary loss function (26).

Centralized Adversarial Environment

Consider a zero-sum game with two players, where player (r) represents the network, and player (c) represents the adversarial environment [11]. The set of feasible strategies for the network is $r \in \{\varnothing, R\}$ and the set of feasible strategies for the environment is $c \in C$. The matrix of payoffs made by the network to the environment $L(c,r|w)$ is given by (10). According to this game theoretic framework, the optimal network strategy $r \in \{\varnothing, R\}$ represents the admission and routing strategy guarding against the worst case scenario with respect to the route costs $c \in C$. The value of the game $V = V(w|C,R)$ represents the expected performance loss due to the admission and routing decisions $r \in \{\varnothing, R\}$ for a single request under incomplete information on the implied costs of the resources $c \in C$ selected by adversarial environment. In a particular case when the payoff function $L(c,r|w)$ has a saddle point, i.e., $L_{\min}^{\max} = 0$, the environment and the network have pure optimal strategies and the value of the game is $V = 0$. In a case when the payoff function $L(c,r|w)$ does not have a saddle point, i.e., $L_{\min}^{\max} > 0$, the environment and the network have mixed optimal strategies which are probability distributions on $c \in C$ and $r \in \{\varnothing, R\}$ respectively, and the value of the game is $V > 0$.

Note, that according to the centralized scenario the environment is free to assign non-zero probabilities to all possible 2^L combinations of binary variables $d_l \in \{0,1\}$, i.e.,

$$(d_l : l \in R) \in \{0,1\}^L \tag{29}$$

Decentralized Adversarial Environment

In the "decentralized" scenario the environment is capable of random selection of each binary variable $d_l \in \{0,1\}$, but selections for different links are independent from each other:

$$\Pr\{d_l : \forall l\} = \prod_{r \in R} b_l^{d_l} (1 - b_l)^{1-d_l} \tag{30}$$

where

$$\Pr\{d_l\} = \begin{cases} b_l & if \quad d_l = 1 \\ 0 & if \quad d_l = 0 \end{cases} \tag{31}$$

This scenario can be modeled as a non-cooperative game of $K+1$ players [11]. Player (r) with a set of feasible strategies $r \in \{\varnothing, R\}$ and utility function $-L(c,r|w)$ represents the network. A player (d_l) with a set of feasible strategies $d_l \in \{0,1\}$ and utility function $L(c,r|w)$ represents a player selecting the cost of the link l. The network makes admission and routing decisions $r \in \{\varnothing, R\}$ with probabilities $a_r \geq 0$, $\sum a_r = 1$, where the admission probability is

$$a_\Sigma = \sum_{r \in R} a_r \tag{32}$$

and the rejection probability is $a_\varnothing = 1 - a_\Sigma$. A player (d_l) randomly selects the binary variable d_l with probabilities (31). Selections by all players are independent from each other, i.e., (30) holds. This selection results in the average loss

$$\bar{S}(b,a) = \sum_r a_r \sum_d S(d,r) \prod_l b_l^{d_l} (1 - b_l)^{1-d_l} \tag{33}$$

The non-cooperative (Nash) equilibrium is determined by solution to the following optimization problem [11]:

$$V = \bar{S}(b^*, a^* | w) = \min_a \max_b S(b, a | w) \tag{34}$$

subject to constraints

$$\sum a_r = 1,\ a_r \geq 0,\ r \in \{\varnothing, R\} \tag{35}$$

$$0 \leq b_l \leq 1,\ \forall l \tag{36}$$

According to this game theoretic framework, the optimal network strategy selectas route $r \in R$ with probability a_r^*. The optimal admission probability is $a_\Sigma^* = \sum a_r^*$, $r \in R$. The optimal average loss due to uncertainty is represented by (34).

5. SOLUTION FOR A SYMMETRIC CASE

In this section we explicitly identify the optimal network strategies by solving the corresponding games in a case of parallel homogeneous structure when costs of all feasible routes lie within the same confidence interval:

$$c_r \in [\breve{c}, \hat{c}] \tag{37}$$

In this case the network has two pure admission strategies: to accept or reject an arriving request. Our goal is to find the optimal admission probability a_Σ^*. Once admitted, the request is carried on a route r selected equiprobably from the set of K feasible routes: $a_r^* = a_\Sigma^* / K$, $r \in F$.

Centralized Adversarial Environment

Since a case of a single feasible route $K = 1$ is covered in [7], in this and next subsections we assume that $K \geq 2$. We consider $K+1$ pure strategies for the malicious environment. Strategy s_k, $k = 0,..,K$ assigns $c_r = \hat{c}$ for a set of k feasible routes $r \in \{r_i : i = i_1,..,i_k\}$, randomly, with equal probabilities selected from $F = \{r_1,..,r_K\}$ and assigns $c_r = \breve{c}$ for routes $r \in F \setminus \{r_i : i = i_1,..,i_k\}$. The pay-off matrix (19) takes the form of the following $(K+1)\times 2$ matrix $L = (L_{kj})_{k,j=0}^{K,1}$:

$$\begin{array}{ccc} & reject & accept \\ s_k: & L_{k0} & L_{k1} \end{array}$$

where the risk associated with rejection is

$$L_{k0} = \begin{cases} \max\{0, j(w - \breve{c})\} & if\ k = 0,..,K-1 \\ \max\{0, j(w - \hat{c})\} & if \quad k = K \end{cases}$$

the risk associated with admission is

$$L_{k1} = \begin{cases} \max\{0, j(w - \breve{c})\} - j_k & if\ k \leq K-1 \\ -\min\{0, j(w - \hat{c})\} & if \quad k = K \end{cases}$$

and the expected utility of servicing a request of cost w, given the environment strategy s_k, is

$$j_k = \frac{k}{K} j(w - \hat{c}) + \left(1 - \frac{k}{K}\right) j(w - \breve{c})$$

A $(K+1)\times 2$ game can be solved explicitly [11]. It is easy to verify that this game always has a saddle point if $K \geq 2$. The optimal admission probability is

$$a^*_\Sigma = \begin{cases} 0 & if\ w < w^* \\ 1 & if\ w > w^* \end{cases}$$

where $w = w^*$ is the unique solution to the following equation:

$$j(w - \breve{c}) + (K-1) j(w - \hat{c}) = 0 \qquad (38)$$

The optimal strategy for the environment is $s^{opt} = s_{k^*}$ where

$$k^* = \begin{cases} K & if\ w < \breve{c} \\ K-1 & if\ w > \breve{c} \end{cases}$$

The value of the game is

$$V = \begin{cases} 0 & if & w \leq \breve{c} \\ j(w - \breve{c}) & if & \breve{c} < w \leq w^* \\ j(w^* - \breve{c}) & if & w > w^* \end{cases}$$

For utility function (12) equation (38) can be solved explicitly:

$$w^* = \frac{1}{g} \log_e \left[\frac{\exp(g\breve{c}) + (K-1)\exp(g\hat{c})}{K} \right]$$

In a particular case of a linear utility function $j(x) \equiv x$ equation (38) yields:

$$w^* = \frac{\breve{c} + (K-1)\hat{c}}{K}$$

Decentralized Adversarial Environment

In a case of question the average loss (24) takes the following form:

$$\bar{S}(b, a | w) = (1 - b^K)\max\{0, j(w - \breve{c})\} +$$
$$b^K \max\{0, j(w - \hat{c})\} - a\left[(1-b) j(w - \breve{c}) + b j(w - \hat{c})\right]$$

where $b_r = b$, $\forall r \in R$. It can be shown that the solution to the optimization problem (34)-(36) is as follows. The optimal admission probability is

$$a^*_\Sigma = \begin{cases} 0 & if\ w < w^* \\ 1 & if\ w > w^* \end{cases}$$

and the optimal strategy for the environment is

$$b^* = \begin{cases} 0 & if & w \leq w^* \\ \left[\dfrac{j(w - \breve{c}) - j(w - \hat{c})}{K j(w - \breve{c})} \right]^{\frac{1}{K-1}} & if & w^* < w \leq \hat{c} \\ (1/K)^{1/K} & if & w > \hat{c} \end{cases}$$

where $w = w^*$ is the unique solution to the following equation:

$$j(w - \hat{c}) + \left(K^{1/K} - 1\right) j(w - \breve{c}) = 0 \qquad (39)$$

For utility function (12) equation (39) can be solved explicitly:

$$w^* = \frac{1}{g} \log_e \left[\frac{\exp(g\hat{c}) + (K^{1/K} - 1)\exp(g\breve{c})}{K^{1/K}} \right]$$

In a particular case of a linear utility function $j(x) \equiv x$ equation (39) yields:

$$w^* = \frac{\hat{c} + (K^{1/K} - 1)\breve{c}}{K^{1/K}}$$

6. CONCLUSIONS

This paper proposes a framework for performance evaluation and optimization of randomized routing. The framework is based on interpreting randomized routing as a regularized solution to the route cost minimization problem. Special attention is paid to the game theoretic framework for the regularization, intended to guard against the worst case scenario with respect to uncertain parameters lying within known "confidence" regions. According to this game-theoretic framework the optimal network strategy is identified with the Nash equilibrium in the corresponding game. The paper explicitly identifies the optimal strategy in a case of parallel, homogeneous structure under various assumptions on the ability of adversarial environment to manipulate the link costs. Future efforts should be directed towards developing computationally feasible solutions for the corresponding games for more realistic topologies.

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